Exercises

1. Find all divisors of 20.

2. Find all divisors of 29,601.

3. Find all divisors of 1.

4. Find the quotient and remainder for a divided by d when

   (a) $a = 281$ and $d = 23$.
   (b) $a = 281$ and $d = 12$.
   (c) $a = 291$ and $d = 23$.
   (d) $a = 291$ and $d = 12$.

5. Find the quotient and remainder for $10^k + 1$ divided by 11 for $k = 1, 2, 3, 4, 5$.

6. Compute the m-adic representation of 526 for $m = 2, 3, 7$, and 9.

7. Compute the 100-adic representation of 783,614,955.

8. Prove that n is even, then n^2 is divisible by 4.

9. Prove that n is odd, then $n^2 - 1$ is divisible by 8.

10. Prove that $n^3 - n$ is divisible by 6 for every integer n.

11. Prove that if d divides a, then d^k divides a^k for every positive integer k.

12. Prove that if d divides a and d divides b, then d divides $ax + by$ for all integers x and y.

13. Prove that if a and d are integers such that d divides a and $|a| < d$, then $a = 0$.

14. Prove that divisibility is transitive, that is, if a divides b and b divides c, then a divides c.

15. Prove by induction that $n \leq 2^{n-1}$ for all positive integers n.

16. Prove by induction that

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

for all positive integers n.